The Hollomon n – value, and the strain to necking in steel

General

It is well documented that the true strain to necking, ε_n, in a uniaxial tensile test provides a valuable measure of the stretch formability of a material (1) – the stretch formability increasing with increasing ε_n.

Quite frequently experimentally recorded σ-ε curves are described by the Hollomon – equation (2)

$$\sigma = K \cdot \varepsilon^n \qquad (1)$$

where σ is the true stress, K a material constant, ε is the true strain and n is the strain hardening index or the n-value, and since necking commences at maximum load, it holds that

$$\varepsilon_n = n \qquad (2)$$

Hence, if the σ-ε curve of a material can be represented by eqn(1) then it also holds that n is a measure of the stretch formability of the material. In fact, since n is readily determined from the slope of a plot of log σ versus log ε, the latter parameter has frequently been used for this purpose. Unfortunately, eqn(1) very seldom gives an accurate description of experimental σ – ε data. Instead, it seems that n is strain dependent and that the constant value of n that is often reported is merely an "average" taken over some specific strain interval on a log σ-log ε plot. A pronounced non-linearity of the experimentally-determined log σ-log ε relationship for e.g. iron and steel has also been reported (3). This so-called "double-n" behaviour is usually represented by two straight lines intersecting at some strain ε_1 and such that

$$\sigma = K_1 \cdot \varepsilon^{n_1} \qquad \varepsilon_{lu} \le \varepsilon \le \varepsilon_1 \qquad (3a)$$

$$\sigma = K_2 \cdot \varepsilon^{n_2} \qquad \varepsilon_1 \le \varepsilon \le \varepsilon_n \qquad (3b)$$

where K_1, K_2, n_1 and n_2 are constants and ε_{lu} is the Lüders strain. In this case it is similarly reasonable to assume that the two n-values are averages taken over the strain intervals (ε_{lu}-ε_1) and (ε_1-ε_n). It seems therefore that the strain hardening index as defined in the Hollomon relationship is an uncertain parameter and hence that its application as a measure of e.g. the stretch formability of steel sheet must in many instances be questioned.

It is the objective of this of this paper to discuss the physical significance of n and to show that the true strain to necking, ε_n, as determined by the present dislocation model for bcc metals represents a more reliable measure of e.g. the stretch formability of steel. It also leads to increased possibilities to identify individual effects of the parameters defining the strain to necking and, as a result of this, to optimise the properties for stretch forming.

It is the objective of this paper to present and discuss some of these possibilities.

Application of the Bergström bcc-model to the strain hardening index n.

According to Bergström(4) the stress – strain behaviour of bcc – metals may be written

$$\sigma = \sigma_{i0} + \alpha \cdot G \cdot b \cdot \left(\frac{m}{\Omega \cdot b \cdot s_0} \cdot (1 - e^{-\Omega \cdot \varepsilon}) + \rho_0 \cdot e^{-\Omega \cdot \varepsilon} \right)^{\frac{1}{2}} \quad (4)$$

where σ_{i0} is the friction stress, α is a dislocation strengthening factor, G is the shear modulus, b is the nominal value of the Burgers vector, m is the Taylor factor, Ω is the dislocation remobilisation factor, s_0 is the mean free path of dislocation motion and ρ_0 is the "grown-in" dislocation density.

By introducing the parameter U_0 for dislocation generation

$$U_0 = \frac{m}{b \cdot s_0} \quad (5)$$

eqn(4) may be written

$$\sigma = \sigma_{i0} + \alpha \cdot G \cdot b \cdot \left(\frac{U_0}{\Omega} \cdot (1 - e^{-\Omega \cdot \varepsilon}) + \rho_0 \cdot e^{-\Omega \cdot \varepsilon} \right)^{\frac{1}{2}} \quad (6)$$

In order to obtain an analytical expression for n in terms of the parameters in eqn(6) we may proceed in the following way. From eqn(1) we obtain after differentiation

$$\frac{d\sigma}{d\varepsilon} = \frac{n \cdot \sigma}{\varepsilon} \quad (7)$$

By differentiation of eqn(6) with respect to strain and comparison with eqn(7), we obtain the following expression for n

$$n = \frac{1}{2} \cdot \frac{\varepsilon \cdot e^{-\Omega \cdot \varepsilon}}{1 + \frac{\sigma_{i0}}{\sigma_d(\varepsilon)}} \cdot \frac{U_0}{\rho(\varepsilon)} (1 - \frac{\rho_0}{\rho_{max}}) \quad (8)$$

where

$$\rho(\varepsilon) = \frac{U_0}{\Omega} \cdot (1 - e^{-\Omega \cdot \varepsilon}) + \rho_0 \cdot e^{-\Omega \cdot \varepsilon} \quad (9a)$$

$$\sigma_d(\varepsilon) = \alpha \cdot G \cdot b \cdot \sqrt{\rho(\varepsilon)} \quad (9b)$$

$$\rho_{max} = \frac{U_0}{\Omega} \quad (9c)$$

Here ρ_0/ρ_{max} may usually be neglected for well-annealed specimens.

It is clear from eqn(8) that n varies monotonically with strain and in order to further elucidate this behaviour we will make some comparisons with experimental data.

Experiments

In the experiments a H_2-treated steel is used in order to avoid a Lüders strain. Hence a maximum strain interval can be used for the analysis. The specimens are tensile tested at 25C,400C and 620C at a strain rate of 10^{-3} s^{-1}.

Fitting procedure

A special Matlab subroutine, based on the Matlab Curve Fitting Toolbox, is designed for the purpose of this study. and eqn(4) is fitted to the experimental true stress – true strain curves. In the fitting procedure the following parameters are kept constant, see Table 1:

α	1	1	1
G, MPa	78500 (25C)	69700 (400C)	60800 (620C)
b, m	$2.5\cdot10^{-10}$	$2.5\cdot10^{-10}$	$2.5\cdot10^{-10}$
m	2	2	2

Table 1. Constant parameter values

The parameters Ω, σ_{i0}, ρ_0 and s_0 are allowed to vary freely, within realistic limits, until the best fit is obtained. The results from the fitting of eqn(4) to the stress-strain curve recorded at 25C is shown in Fig. 1. In the upper left graph the fitted stress-train curve is shown – this curve (red) is covering the experimental data. The red cross indicates the calculated strain to necking and corresponding flow stress. To the upper right we see the corresponding theoretical ρ-ε curve. To the lower left the parameter values recorded in the fit are shown while the figure to the lower right gives the errors in the fit. We see that the average error e_1 is smaller than 0.3 MPa while the running average error e_2 is approximately equal to 0.003 MPa. For a more detailed presentation of the fitting of eqn(6) to experimental data, see Paper 1 on this homepage.

The parameter values obtained in fitting eqn(4) to the true stress-strain curves recorded at the temperatures 25C , 400C and 620C are presented in Table 2.

T C	25C	400C	620C
Ω	4.59	7.92	12.3
$\sigma_{i0,}$ (MPa)	27.87	24.81	30.85
ρ_0 (m^{-2})	$9.6\cdot10^{11}$	$8.7\cdot10^{11}$	$9.5\cdot10^{11}$
$s_{0,}$ m	$4.35\cdot10^{-6}$	$7.08\cdot10^{-6}$	$21\cdot10^{-6}$
G, MPa	78500	69700	60800

Table 2. Parameter values obtained in the fitting procedure

The strain dependence of n

It is generally assumed in fitting the Hollomon relationship, see eqn(1), to experimental stress-strain data, that the parameters K and n are strain independent. The fact that a "double-n", see eqn(3), and sometimes even a "triple-n" behaviour is observed indicates that this might be a strong simplification. We will now use the present theory, see eqn(8), to demonstrate the actual strain dependence of n for a H2-treated steel strained at the three different temperatures indicated above. In doing this we will use the parameter values obtained in fitting of eqn(4) to the corresponding stress-strain curves, see Table 2. The result is presented in Fig 2. It can be seen that n starts at a low value at small strains and increases to a maximum at ca 2-3% of strain. Thereafter n decreases with increasing strain. It is also evident that n at all strains decreases with increasing temperature which mainly is an effect of an increasing mean free path s and an increasing Ω-value.

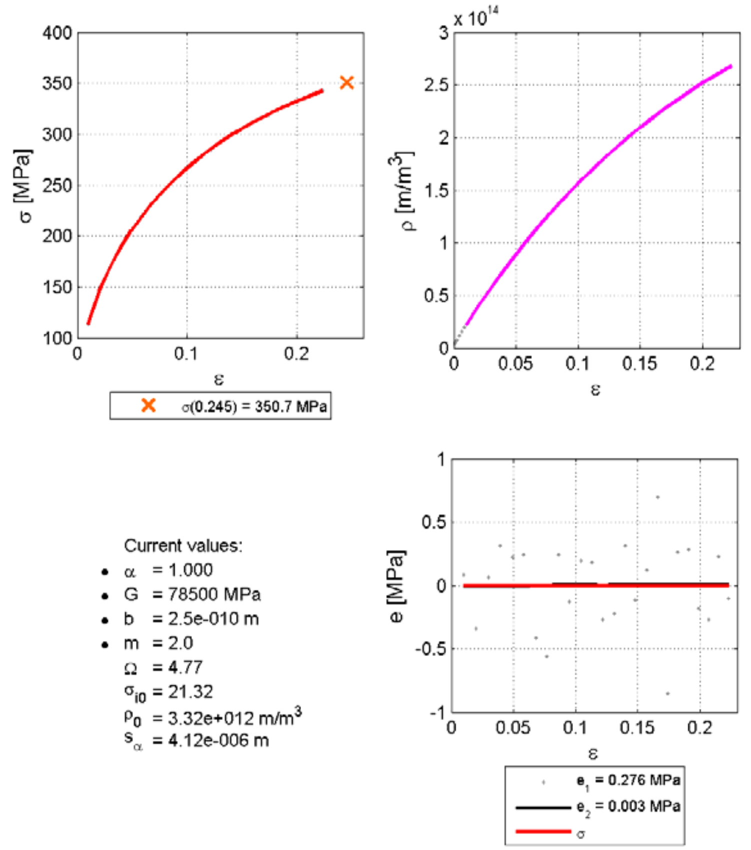

Fig.1. Results from fitting eqn(6) to a room temperature stress-strain curve, see text.

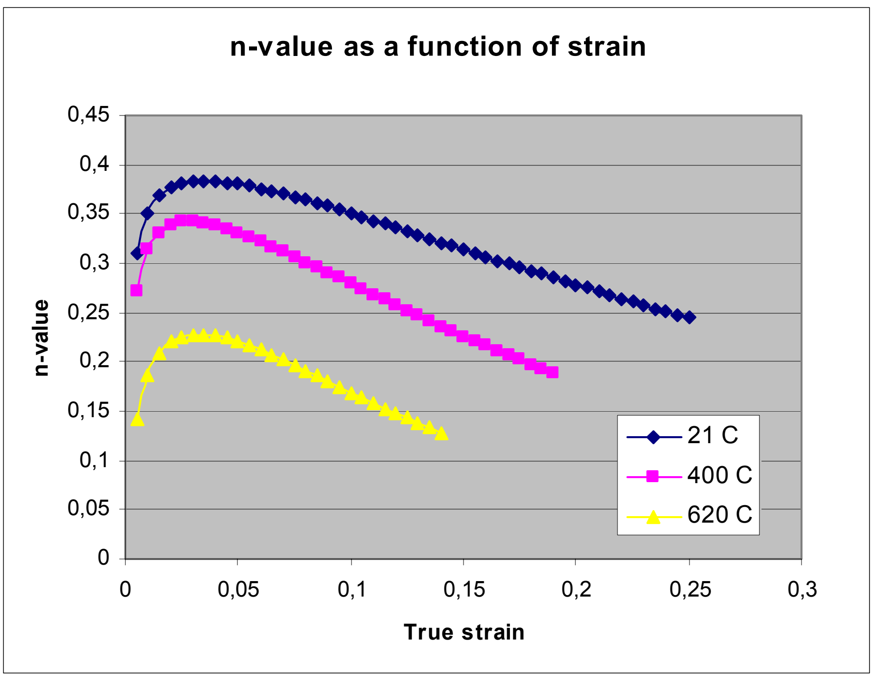

Fig. 2. The n-value in the Hollomon equation as a function of strain for a H_2-treated steel tested at 25C, 400C and 620C and at a strain-rate of 10^{-3} s^{-1}

Now, in order to elucidate this further we will plot log σ versus log ε for the three temperatures and investigate weather the plots are linear or not. It is quite obvious, and in accordance with the n-ε plots in Fig.2 that, in fact, a triple-n behaviour is prevailing for the three curves, see Fig.3. This is demonstrated in more detail in Fig.4 where the log σ-log ε plot of the stress-strain curve recorded at 25C is approximated by three straight lines. The equations for the three lines are presented in the figure. (The points represent the experimental values).

In the present case we are dealing with a H_2-treated steel where the Lüders band effect has been eliminated. Conventional steels, however, exhibit Lüders strains of several percent and if

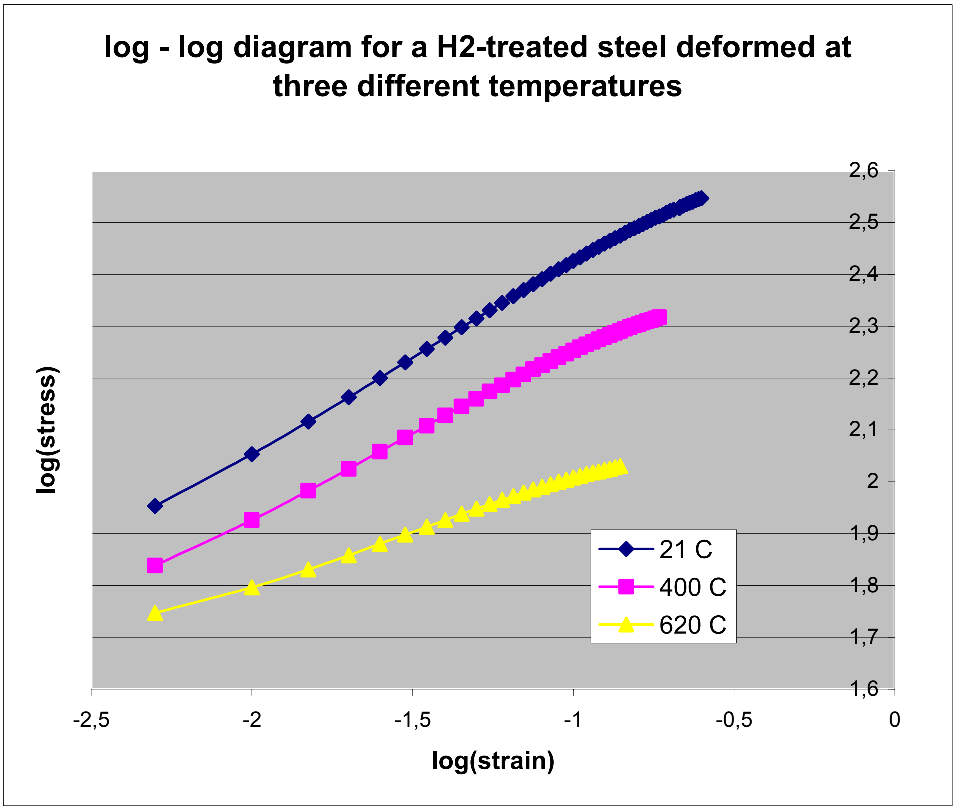

Fig.3. Log σ-log ε plots for a H_2-treated steel strained at 25C, 400C and 620C and at a strain-rate of 10^{-3} s^{-1}.

we exclude the first three to four points of the curves in Fig.3 we see that a double-n behaviour is exhibited which also is typical for conventional steels, see also Fig.4.

From the above results we may therefore conclude that the strain hardening index n in iron and steel varies with strain and that an accurate description of experimental σ-ε curves normally cannot be obtained by the simple slightly modified Eqns (2) and (3). These equations may, however, be used as approximations. The basic problem with equations like the Hollomon ones is, however, that the parameters involved are lacking a physical meaning.

A consequence of this is that these equations are less suitable for a physical analysis of the mechanical properties of metals as well as for a physically based development of their mechanical properties. However, in spite of these problems the n-value is commonly used for judging the mechanical properties of metals and alloys. The basic reason for this is that the n-vales have a long history and that they are well known in the metal as well as in the car industries. Let us, however, hope that the n-values soon will be replaced by physically more realistic measures. Since the n-value in the Hollomon equation lacks a physical meaning and is strain dependent, it also holds that the n-value as a measure of stretch formability is a dubious one.

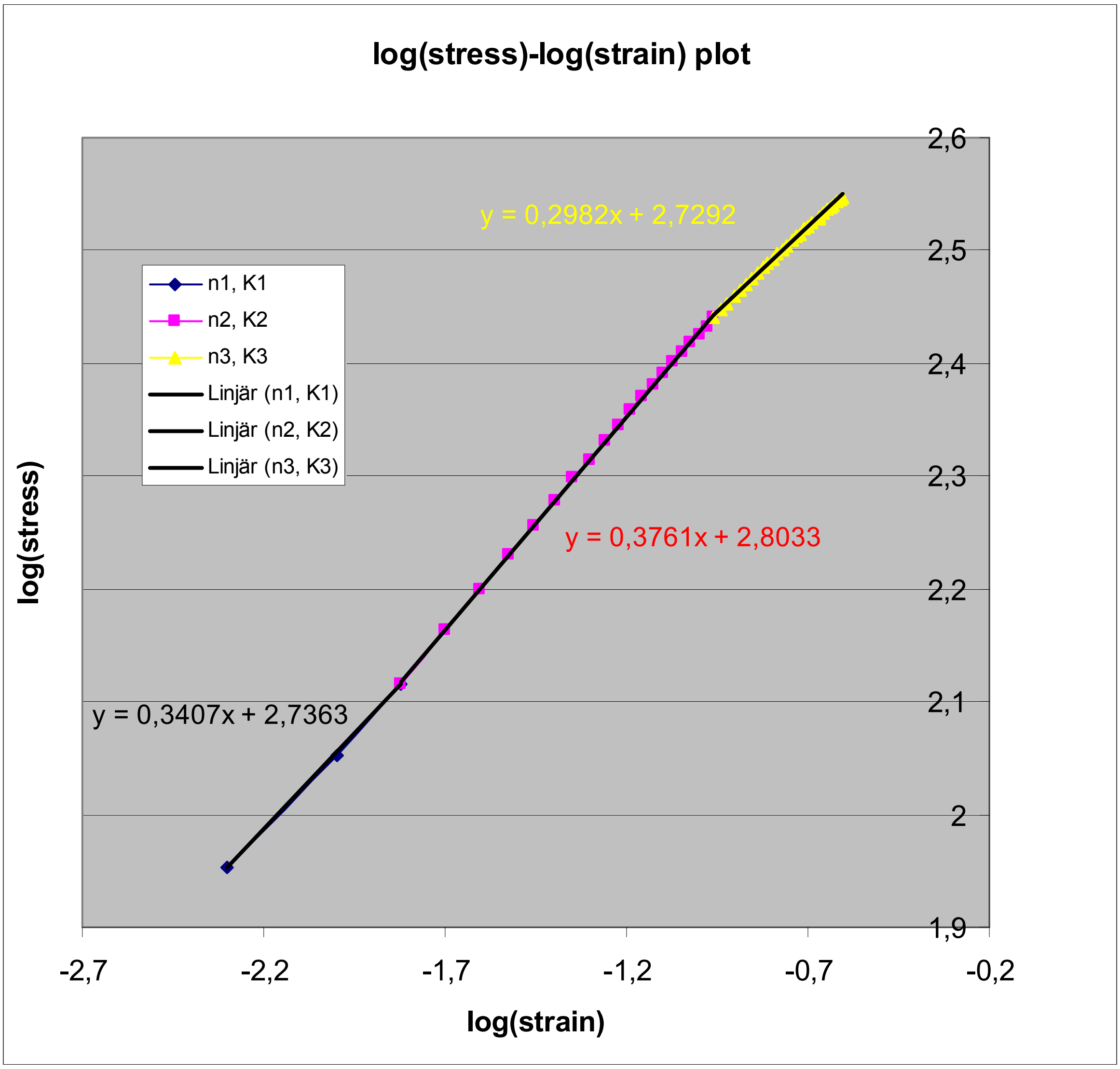

Fig.4. A log σ – log ε plot of the H2-treated steel strained at 25C and at a strain rate of $10^{-3}\ s^{-1}$. The triple-n behaviour is illustrated by linear approximations in black, red and yellow. From the figure we can see that the slope at low strains is equal to approximately 0.34, at intermediate strains ~ 0.38 and at high strains ~0.3.

The true strain to necking, ε_n

An alternative and better way to go is therefore to use the true strain to necking (5). We will therefore demonstrate the advantages with such a procedure by proceeding from eqn(6) and calculate the true strain to necking and to study, on a physical basis, the effects of the rate of dislocation generation, dislocation re-mobilisation, friction stress, etc. on the true strain to necking.

Since necking commences at maximum load, the criterion for plastic instability at uniaxial straining may be written

$$\frac{d\sigma(\varepsilon)}{d\varepsilon} = \sigma(\varepsilon) \tag{10}$$

Now, proceeding from eqns.(6) and (10) and assuming that the "grown-in" dislocation density is small and may be neglected, the following expression for the strain to necking may be derived

$$\varepsilon_n = -\frac{1}{\Omega}\cdot\ln\left[\frac{1}{1+\Omega/2} - \frac{1}{2}\left(\frac{\sigma_{i0}}{\sigma_{d\max}}\cdot\frac{1}{1+\Omega/2}\right)^2 + \frac{\sigma_{i0}}{\sigma_{d\max}}\cdot\frac{1}{1+\Omega/2}\cdot\sqrt{\frac{1}{4}\cdot\left(\frac{\sigma_{i0}}{\sigma_{d\max}}\cdot\frac{1}{1+\Omega/2}\right)^2 + \frac{\Omega/2}{1+\Omega/2}}\right]$$

(11)

where

$$\sigma_{d\max} = \alpha\cdot G\cdot b\cdot\left(\frac{U_0}{\Omega}\right)^{\frac{1}{2}} \tag{12}$$

Unfortunately, this expression is comparatively complicated, but it may be considerably simplified for the low carbon sheet steels normally used for stretch forming.

For the latter type of steel the friction stress, σ_{i0}, is usually smaller than 100 MPa and the value of σ_{dmax} larger than 300 MPa. The value of Ω at the ambient temperature for this type of steel is approximately 5. It thus holds that the quadratic terms in eqn(11) are much smaller than the other terms and that the approximation ln(1+x) ~ x can be used and eqn(11) may be rewritten in the following (approximate) form

$$\varepsilon_n \approx \frac{1}{\Omega}\cdot\ln\left(1+\frac{\Omega}{2}\right) - \frac{1}{\Omega}\cdot\frac{\sigma_{i0}}{\sigma_{d\max}}\cdot\sqrt{\frac{\Omega/2}{1+\Omega/2}} \tag{13}$$

The friction stress, σ_{i0}, has the following components.

$$\sigma_{i0} = \sigma_g + \sigma_p + \sigma_s + \sigma^* \tag{14}$$

where σ_g stands for grain size hardening, σ_p for precipitation hardening, σ_s for solution hardening and σ^* for thermal hardening.

It is obvious, therefore, from eqn(13) that a large strain to necking, that is a good stretch formability, is obtained by minimising the four friction stress hardening components in eqn(14) and maximising, $\sigma_{d\,max}$, that is the rate of deformation hardening.

Eqn(13) also indicates that a decrease in Ω, i.e. a decrease in the rate of dislocation remobilisation, results in an increased strain to necking. Since there is a strong effect of Ω on ε_n, it would be of great interest to find measures to make the Ω-value smaller by e.g. processing, alloying or other ways. In the steel studied in the present investigation Ω is equal to 4.6 and the strain to necking is calculated to be 0.245. If it would be possible, by various types of treatments, to make Ω attain half that value, i.e 2.3, while the other parameters were kept constant, then, according to eqn(13), ε_n~0.32. This would imply a dramatic improvement of the ductility and the stretch formability of this type of steel.

Now, the strain to necking may also be calculated directly from eqn(10) by using eqn(6) and the parameter values obtained in the fitting procedure. This can be seen in Fig.1 where the red cross represents the true strain to necking and the corresponding flow stress. Proceeding from such a procedure it is also easily done to investigate the influence of variations in the different parameters in eqn(6) on ε_n. An example of such an analysis is presented in Fig.5 where the effect of variations in the friction stress, σ_{io}, is studied. In this figure the parameter values obtained in the fit of the stress-strain curve recorded at 25C are used as base values. It can be seen in the figure that a decrease in σ_{i0} from 150 MPa to10 MPa results in an increase of the strain to necking from 20% to 25%.

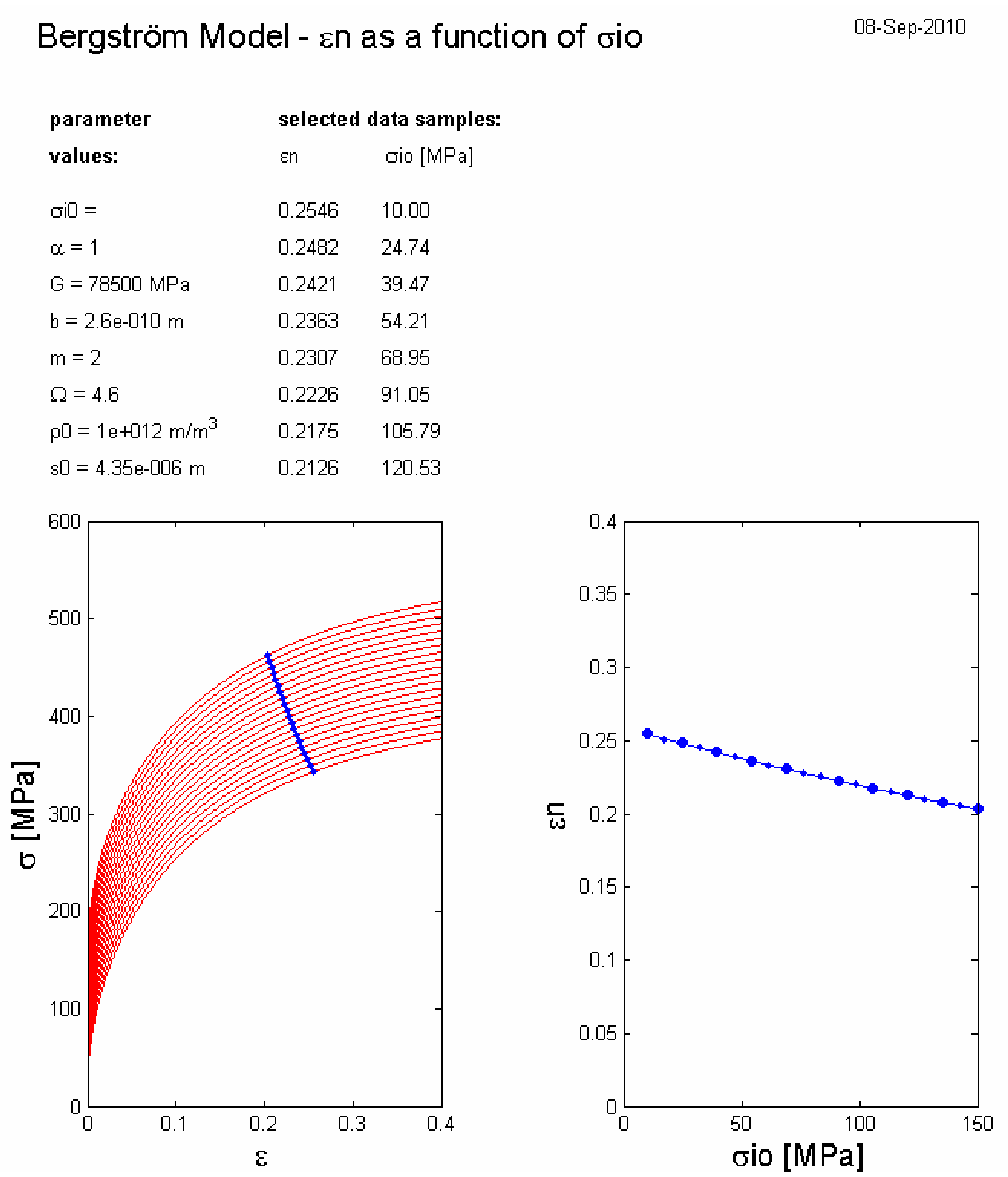

Fig.5. Variations in ε_n as a function of σ_{i0} for a H2-treated steels tested at 25C

Discussion
In the present study we have used Bergström original dislocation theory to study the n-value in the Hollomon equation. It is demonstrated that n is not a constant, as proposed in the empirical Hollomon equation, but in fact strain dependent. This strain dependence also explains the “double-n” and “triple-n” behaviours often reported in the literature.

It is also quite clear from the reasoning above that the n-value is an unreliable parameter for judging the mechanical properties of steel and other metals and alloys. It is certainly better to use the strain to necking. We have therefore, as an example, used Bergströms model to derive an expression relating the strain to necking to physical parameters as friction stress, deformation hardening (mean free path of dislocation motion) and dislocation remobilisation as defined in this theory. Proceeding from such an analysis it is possible to evaluate the individual influence of the various material parameters on the strain to necking and thus stretch formability.

We have also shown that larger ε_n-values for steel may be obtained by decreasing the friction stress and by increasing the rate of work hardening, i.e. decreasing the mean free path of dislocation motion, s. Another effective way of obtaining much larger strains to necking and thus a better stretch formability would be to find methods to reduce the value of the dislocation remobilisation constant, Ω. Let us present a short discussion related to that possibility.

The ability for a dislocation to remobilise depends on several factors besides melting temperature, testing temperature and strain rate. It is well known that the stacking fault energy, γ, plays an important role. A decrease in γ implies that the width of the stacking fault increases and hence that the probability for dislocation cross slip diminishes and consequently that the probability for remobilisation is reduced. Another factor influencing the probability for cross slip is the number of available slip systems. The bcc structure exhibits a large number of slip systems and would therefore show higher Ω-values than the fcc- and hcp-structures. This is in good agreement with experimental observations. While the Ω-value for ferrite at room temperature and a moderate strain rate is approximately equal to 5 the corresponding value for an austenitic steel is approximately 0 and that for cupper approximately 2. It is reasonable to assume that also texture may have an impact on the Ω-value. Also the locking of immobilised dislocations by solute atoms and other configurations of atoms and vacancies may reduce the probability for remobilisation. If we could find a method to reduce the Ω-value for alpha iron to a value close to zero we would have the possibility to reach a ε_n-value close to 0.5 if we have σ_{i0}~0 MPa. We would then have reached the condition for Taylors original theory for work hardening with $\sigma \sim \varepsilon^{1/2}$ and a superb ductility and stretch formability.

Conclusions
It is concluded that to improve stretch formability in steel, that is to increase ε_n, we may proceed in the following ways:

- decrease the grain size hardening component, σ_g, by increasing the grain diameter.
- decrease the precipitate hardening component, σ_p, by reducing the content of precipitates

- decrease the solution hardening component by reducing the amount of atoms in solution
- carry out the forming operations at low enough deformation rates and moderate temperatures. It should be remembered that dynamic strain ageing effects may occur at as low temperatures as 150 – 200C and dramatically change the behaviour.
- try to find methods by which the dislocation remobilisation constant Ω may attain values much smaller than 5 for alpha iron.

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