A THEORY FOR THE TEMPERATURE AND STRAIN-RATE DEPENDENCES OF DISLOCATION RE-MOBILISATION

Background

In PAPER 1of this set of presentations(1): “A dislocation model for the plastic deformation of single-phase alpha-iron - a résumé” it is shown that the temperature dependence of the re-mobilisation parameter Ω may be written

$$\Omega = \Omega_0 + \Omega(T, \dot{\varepsilon}) \qquad (1)$$

where Ω_0 is the athermal component and $\Omega(T, \dot{\varepsilon})$ the a thermal one, see Fig.1.

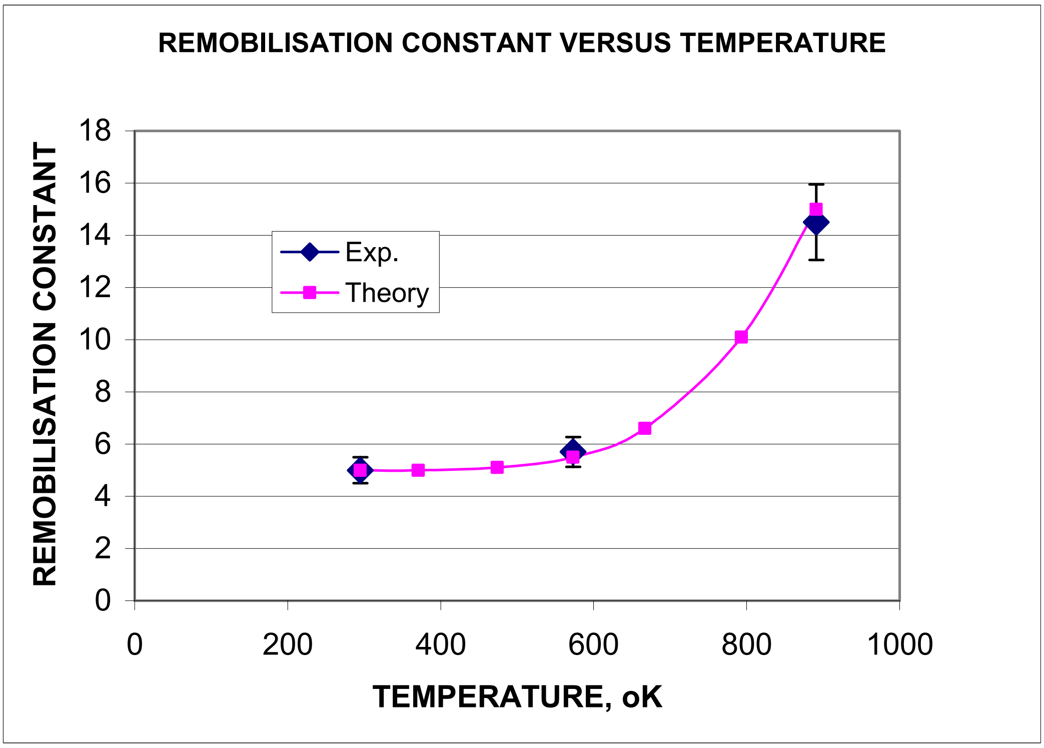

Fig 1. The re-mobilisation parameter Ω as a function of temperature for a Ti – stabilised steel

A theory for $\Omega(T, \dot{\varepsilon})$

Let us start by deriving a simple theory for $\Omega(T, \dot{\varepsilon})$ by making the following assumptions(2):

1. The re-mobilisation of immobile dislocations takes place predominantly in the dislocation cell walls – this is supported by in-situ HVEM studies.
2. The re-mobilisation process is controlled by vacancy climb of the immobile dislocations – this is a reasonable assumption inter alia in the light of the fact that a supersaturation of vacancies rapidly develops in the material on straining.
3. During the course of deformation the vacancies diffuse towards the dislocation cell walls where they are absorbed by climbing dislocations.
4. The vacancy diffusion is controlled by the Einstein relationship

$$l = \sqrt{2 \cdot D \cdot t} \qquad (2)$$

where l is the average distance moved by the vacancies during time, t, and D is the diffusion constant. Since it is assumed that a vacancy supersaturation exists in the matrix D is given by

$$D = D_0 \cdot \exp(-\frac{Q_m}{R \cdot T}) \tag{3}$$

where D_0 is a constant, Q_m, is the vacancy migration energy and R and T have their usual meaning.

Proceeding from assumptions 3 and 4 above we may now write

$$N = 2 \cdot n_0 \cdot l \tag{4}$$

where N is the number of vacancies that have arrived at unit area of the cell walls after time, t, n_0 is the number of vacancies per unit volume and the factor 2 takes into account the fact that vacancies are reaching the cell walls from two opposite directions. Now, since it is assumed that the re-mobilisation of the immobile dislocations is initiated by vacancy climb in the cell walls, points 1 and 2 above, it is also reasonable to assume that $\Omega(\dot{\varepsilon}, T)$ is linearly related to the number of vacancies reaching the cell walls and from eqn(4) we have

$$\Omega(\dot{\varepsilon}, T) = k \cdot n_0 \cdot l(\tau) \tag{5}$$

where k is a proportionality constant and $l(\tau)$ is the average distance moved by the vacancies during the mean time, τ, the immobile dislocations remain in rest between successive re-mobilisations. Combining eqns (5) and (2) yields

$$\Omega(\dot{\varepsilon}, T) = k \cdot n_0 \cdot \sqrt{2 \cdot D \cdot \tau} \tag{6}$$

Now, the following simple expression for τ was derived in section xxxxx

$$\tau = \frac{1}{\Omega \cdot \dot{\varepsilon}} \tag{7a}$$

where $\dot{\varepsilon}$ is the strain rate. Since we are presently only considering thermal re-mobilisation, τ may be written

$$\tau(\dot{\varepsilon}, T) = \frac{1}{\Omega(\dot{\varepsilon}, T) \cdot \dot{\varepsilon}} \tag{7b}$$

and inserting the latter expression into eqn(6) yields

$$\Omega(\dot{\varepsilon}, T) = k \cdot n_0 \sqrt{2 \cdot D} \cdot \sqrt{\frac{1}{\Omega(\dot{\varepsilon}, T) \cdot \dot{\varepsilon}}} \tag{8}$$

Solving for $\Omega(\dot{\varepsilon}, T)$ and replacing D by the expression in eqn(3) finally gives

$$\Omega(\dot{\varepsilon},T) = \ln\left[k \cdot n_0 \cdot \sqrt{2 \cdot D_0}\right]^{2/3} \cdot (\exp - \frac{Q_m}{3 \cdot R \cdot T}) \cdot (\dot{\varepsilon})^{-1/3} \tag{9}$$

The only unknown parameter in this equation is the parameter product, $k \cdot n_0$, and we shall as a first approximation assume that $k \cdot n_0$ almost instantly attains a constant value on plastic deformation. The values of D_0, Q_m and R are tabulated data and T and $\dot{\varepsilon}$ are defined by the experiment.

Comparison with experiment
By taking the logarithms of both sides of eqn(9) the following expression is obtained

$$\ln \Omega(\dot{\varepsilon},T) = \ln\left[k \cdot n_0 \cdot \sqrt{2 \cdot D_0}\right]^{2/3} - \frac{Q_m}{3 \cdot R \cdot T} - \frac{1}{3} \cdot \ln \dot{\varepsilon} \tag{10}$$

The validity of this equation can be tested in the following two ways:

1. The slope of a $\ln \Omega(\dot{\varepsilon},T) - 1/T$ plot is equal to $-Q_m/3R$ and since R is known Q_m can be determined. If the Q_m values thus obtained are reasonable then the theory may be considered as plausible. Often only the activation energy for self-diffusion, Q, is tabulated but according to a thumb-role

$$Q_m \approx \frac{Q}{2} \tag{11}$$

   and this approximation may be used in cases where the Q_m values are not specified.

2. The slope of a $\ln \Omega(\dot{\varepsilon},T) - \ln \dot{\varepsilon}$ plot should, if the theory is correct, be equal to -1/3.

In order to carry out these tests we will proceed from a H_2-treated steel strained at various temperatures in the range of 25^o – 630^oC and at the strain rates of 10^{-4}, 10^{-3} and 10^{-2} s^{-1}. The Ω-values were evaluated by applying the following equation, see PAPER 1,

$$\sigma = \sigma_{i0} + \alpha \cdot G \cdot b \cdot \left[\frac{m}{b \cdot s}(1 - e^{-\Omega \cdot \varepsilon}) + \rho_0 \cdot e^{-\Omega \cdot \varepsilon}\right]^{\frac{1}{2}} \tag{12}$$

where

σ = true stress
ε = true strain
σ_{i0} = friction stress
α = dislocation strengthening constant
G = shear modulus
b = nominal value of the Burgers vector
m = Taylor constant
s = dislocation mean free path
Ω = dislocation re-mobilisation constant
ρ_0 = "grown-in" dislocation density

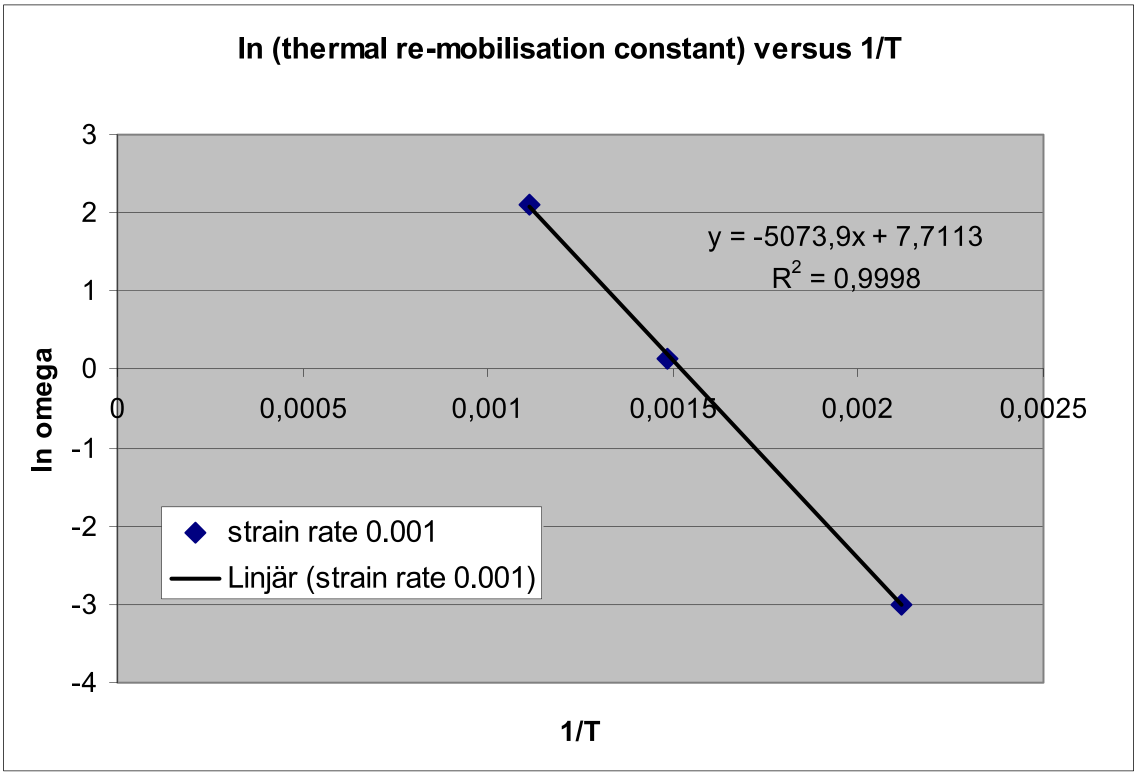

Fig. 1. ln Ω versus 1/T for a H_2 – treated steel at a strain rate of 10^{-3} s^{-1}

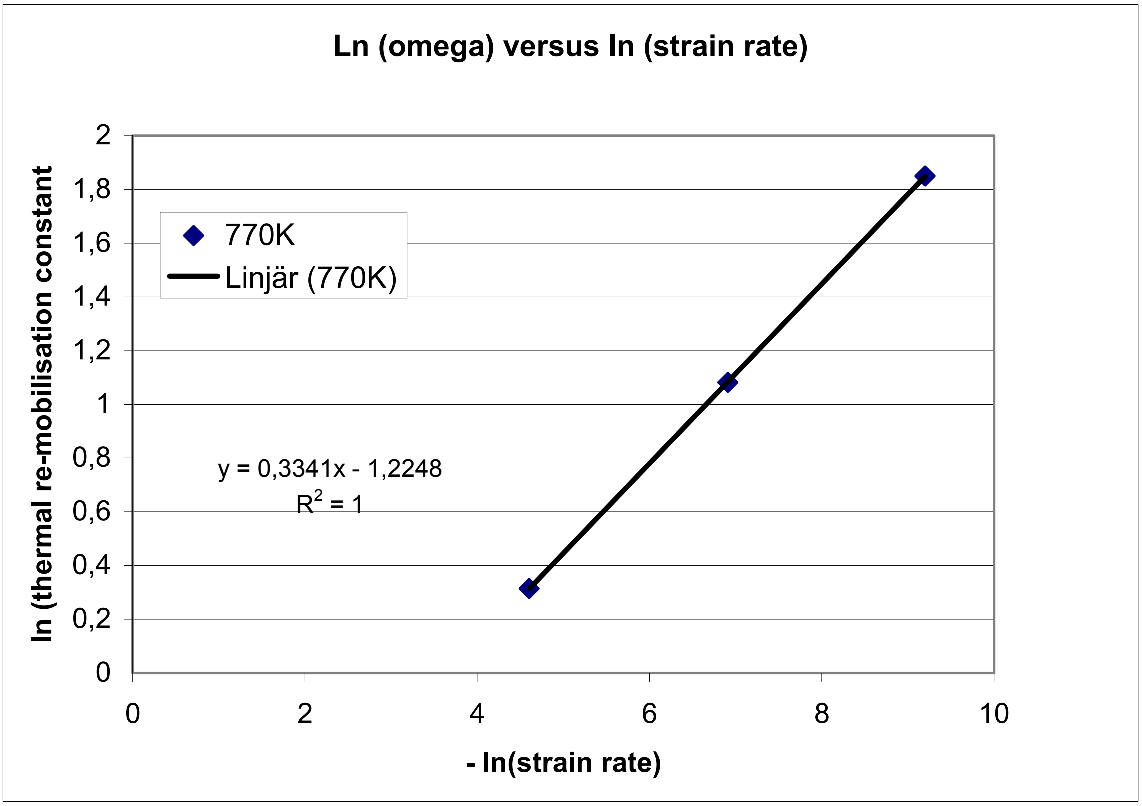

Fig. 2. ln Ω versus ln ε for T = 770K

In Fig.1, a $\ln \Omega(\dot{\varepsilon},T) - 1/T$ plot is shown. The strain rate is 10^{-3} s^{-1} and the three points represent the temperatures 473K, 673K and 900K. It is obvious that a straight line is obtained and from the slope of this line a Q_m – value of 30444 cal/mole can be calculated. According to tabulated data for the self-diffusion of ferrite it holds that (44)

$$D = 2 \cdot e^{\frac{-60000}{R \cdot T}} \quad \text{cm}^2/\text{s} \tag{13}$$

where Q is given in cal/mole. Now, if in agreement with the thumb-rule in eqn(11) we assume that Q(ferrite) $\approx 2 \cdot Q_m$(ferrite), we obtain from eqn(13) that $Q_m \approx$ 30000 cal/mole which is in good agreement with the value of 30444 cal/mole determined theoretically above.

The corresponding $\ln \Omega(\dot{\varepsilon},T) - \ln \dot{\varepsilon}$ plot for the case that T=770K is depicted in Fig 2 and in excellent agreement with the theory a straight slope of 0.334 is obtained.

Now, by using a D_0-value of 2 cm^2/s, see eqn(13) and proceeding from the above results a kn_0 value of $1.85 \cdot 10^{-3}$ is obtained. If this value is considered to be a material independent constant, the following general expression for $\Omega(\dot{\varepsilon},T)$ is obtained.

$$\Omega(\dot{\varepsilon},T) = \left[1.85 \cdot 10^{-3} \cdot \sqrt{2 \cdot D_0}\right]^{2/3} \cdot (\exp - \frac{Q_m}{R \cdot T}) \cdot \dot{\varepsilon}^{-1/3} \tag{14}$$

where D_0 is expressed in cm^2/s and $\dot{\varepsilon}$ in s^{-1}.

It has been demonstrated that eqn(14) may be applied to various types of steel as well as different fcc metals(3)

References

1. Paper 1 on this homepage
2. Y.Bergström, Reviews on powder metallurgy and physical ceramics 2(1983)79-265
3. Y.Bergström, unpublished data